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Successfully combining SUGRA hybrid inflation and moduli stabilisation

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Abstract. Inflation and moduli stabilisation mechanisms work well independently, and many string-motivated supergravity models have been proposed for them. However a complete theory will contain both, and there will be (gravitational) interactions between the two sectors. These give corrections to the inflaton potential, which generically ruin inflation. This holds true even for fine-tuned moduli stabilisation schemes. Following a suggestion by [1], we show that a viable combined model can be obtained if it is the Kähler functions ($G = K + \ln |W|^2$) of the two sectors that are added, rather than the superpotentials (as is usually done). Interaction between the two sectors does still impose some restrictions on the moduli stabilisation mechanism, which are derived. Significantly, we find that the (post-inflation) moduli stabilisation scale no longer needs to be above the inflationary energy scale.

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1. Introduction

Many attempts have been made to implement inflation in extensions of the standard model, although to date there is still no model that is truly convincing. Supersymmetric (SUSY) theories appear to be more promising. They include numerous moduli fields, i.e. scalar fields which in the supersymmetric limit have an exactly flat potential, as is required for slow-roll inflation. Any one of these moduli fields could play the role of the inflaton field. As a concrete example we will consider F-term hybrid inflation in this work. In the SUSY limit it has a flat direction, but when extended to include gravity the situation is less rosy. The large energy density during inflation breaks SUSY spontaneously, and supergravity (SUGRA) effects lift the flatness of the moduli potential. This is the infamous η-problem [2, 3].

Furthermore, the particular form of the SUGRA potential means that all other, non-inflationary sectors of the full theory will couple to the inflation sector. The coupling will be small, in the models we consider it is only of gravitational strength, but it can

nevertheless have large effects. This is a generic problem for all inflation models, and as we will see, this small coupling between the different sectors frequently kills an otherwise good model. As a specific example, we will study the effects of a modulus stabilisation sector on 4D $\mathcal{N} = 1$ SUGRA F-term hybrid inflation. All the other, non-inflationary moduli fields must be fixed during inflation, and so a full SUGRA theory must include additional physics to do this. For this we will consider KKLT-like [4] and KL-like [5] moduli stabilisation schemes. As we will see, the moduli sector gives rise to additional — and quite generically fatal — corrections to the inflaton potential. This raises the questions of whether the original SUSY hybrid inflation model can actually be embedded in a full, realistic theory, and if so, are its original predictions valid? For the answer to both these questions to be yes, the coupling between the two sectors must somehow be minimal, so that neither the moduli corrections to the inflation potential, nor the inflaton corrections to the moduli stabilisation potential ruin the model. As we will show, it is possible, but non-trivial, to achieve this.

There are of course many other models of inflation, which offer alternative approaches to the issue of moduli-inflation coupling. For example, in modular inflation models the modulus field itself is the inflaton [6]. In a sense, the coupling is maximal — nevertheless successful (fine-tuned) models have been constructed [7]. In brane inflation models the inflaton potential arises from brane interactions, and depends explicitly on the volume modulus. Stabilising the modulus field then inevitably gives a curvature correction to the inflation potential [8]. However explicit examples have been constructed where, for fine-tuned parameters, the corrections to η cancel to a high degree, allowing inflation [9, 10]. In contrast to the above models, our strategy is to decouple the inflation and modulus sectors as much as possible. One advantage of this is that it also allows us to decouple the scale of inflation from the gravitino mass scale. At the cost of tuning, it is then possible to have the gravitino in the phenomenologically favoured TeV range without the need for low scale inflation.

The η-problem is a common feature of SUGRA inflation models. To illustrate it, consider a canonically normalised inflaton field with $K = |\phi|^2$. The inflationary potential is of the form $V \sim e^K V_* \sim V_*(1 + |\phi|^2 + \cdots)$, with V_* the nearly constant energy density driving inflation. It follows that the slow-roll parameter $\eta = V''/V$ is of order unity, and slow-roll inflation does not occur. To avoid this conclusion one can fine-tune the model such that the coefficient of the $|\phi|^2$-term in the potential cancels. More elegantly perhaps, one can try to achieve the same using a symmetry. An example of the latter approach is the (accidental) Heisenberg symmetry of the Kähler potential in D-term hybrid inflation [11]. In this paper we avoid the above η-problem by using a shift symmetry for the inflaton, $\phi \to \phi + a$, which leaves the Kähler potential invariant [6, 12]. Since the inflaton field $\mathrm{Re}(\phi)$ no longer appears explicitly in the Kähler potential, the large mass corrections to the inflaton field are avoided.

However, the shift symmetry does not kill all the corrections to the inflaton potential. In the presence of moduli fields η- (and ϵ-) problems appear again. As a concrete example, consider the case of a single modulus field T. If moduli fields are

present, they need to be fixed during inflation. The modulus potential typically has a local minimum at finite field value separated by a barrier from the global minimum at infinity. The classic example is the KKLT potential [4]. To assure that the modulus does not run away during inflation the barrier should be large. This is the case if the modulus mass is large $m_T > H_*$, with H_* being the Hubble constant during inflation [5]. Now since the moduli stabilisation mechanism breaks SUSY, there are soft corrections to the inflaton potential, typically of $\mathrm{O}(m_{3/2}H_*)$. The flatness of the inflaton potential is lost unless the gravitino mass is sufficiently small $m_{3/2} < H_*$. The problem with this is that one cannot tune the gravitino mass arbitrarily: in a generic, KKLT-like potential $m_{3/2} \sim m_T$, and a small gravitino mass is at odds with keeping the modulus fixed. It is therefore difficult to embed inflation in such a scheme.

A solution to the above moduli problem put forward by Kallosh and Linde (henceforth denoted by KL) [13] is to fine-tune the modulus potential so that $m_{3/2} \ll m_T$. Then if the Hubble constant during inflation is between these two mass scales, the modulus remains fixed while the soft corrections to the inflaton mass are small. Such a set-up has the additional advantage that the gravitino mass can be in the TeV range without the need for low scale inflation. KL gave an explicit realisation of this idea using a racetrack potential for the modulus. All problems then appear to be solved, but this is deceiving. Although the moduli corrections are small after inflation thanks to the fine-tuning in the KL set-up, this is not necessarily true during inflation. During inflation the modulus field T is slightly displaced from its post-inflationary minimum, disrupting the minute fine-tuning of the potential, with potentially serious consequences. Indeed, as we will show, in F-term hybrid inflation the effects of the modulus displacement are substantial, resulting in $\eta \approx -3$ and ruining inflation. The need to include the dynamics of the modulus field during inflation was previously noted in [14, 15].

In this paper we will study F-term hybrid inflation, which serves to illustrate all the observations made above. It is a multi-field model of inflation, consisting of the inflaton field, and two oppositely charged waterfall fields which are responsible for ending inflation. When combined with a KKLT modulus sector, the corrections to both the inflaton and the waterfall field potentials are large. Although the mass correction to the inflaton can be protected by a shift symmetry, this is not the case for the waterfall fields, and as a result there is generally no graceful exit from inflation. Tuning the modulus sector, as in the KL set-up, can reduce these corrections to a harmless size. However all of this is under the assumption that the modulus T is fixed during inflation. Taking the modulus dynamics into account we find that even in the fine-tuned KL-stabilisation scheme the corrections are not harmless after all. On the contrary, they prevent inflation from working.

In all previous studies of the effect of the moduli sector on inflation [14, 15, 16, 17], the Kähler and superpotentials of the modulus and inflaton sectors were simply added to get the combined theory, i.e. take $W_{\rm total} = W_{\rm inf} + W_{\rm mod}$ to get the full superpotential. In this paper we instead multiply the superpotentials: $W_{\rm total} = W_{\rm inf} W_{\rm mod}$, as proposed by Achúcarro and Sousa [1]. As we will show, this greatly reduces the moduli corrections.

Indeed F-term hybrid inflation combined with KL, or even KKLT, in this way can give a viable inflation model. Although multiplying superpotentials may sound odd at first, it is natural in a supergravity formulation in terms of the Kähler function $G = K + \ln|W|^2$. Any supersymmetric theory only depends on the Kähler- and superpotential through the combination G, suggesting that it is the only significant quantity. Adding the Kähler functions of the two sectors is equivalent to adding their Kähler potentials and multiplying their superpotentials.

Adding Kähler functions has the nice property that a SUSY critical point of the modulus sector is automatically a SUSY critical point of the full theory as well [1, 18] — this feature is at the heart of the reduced moduli corrections. In the limit of a small gravitino mass, all the corrections to the inflaton potential are small, including those due to the dynamics of the modulus field during inflation. The resulting inflationary model thus gives similar inflationary predictions to the usual F-term hybrid inflation in the absence of a modulus sector. Although there are still some constraints on the model parameters, we want to stress that successful inflation is achieved without the need for fine-tuning — this is in contrast to most other combined inflaton-moduli models. A notable feature of the model is that it is possible for the vacuum modulus mass to be smaller than the Hubble scale during inflation, without the modulus running off to infinity.

This paper is organised as follows. In the next section we provide the relevant background material. We start with a short review of standard F-term hybrid inflation, both in a SUSY and SUGRA theories. This is followed by a concise discussion of moduli stabilisation in KKLT- and KL-style schemes. In section 3 we discuss the resulting model when the two sectors are combined by adding superpotentials. As we will see, even in the fine-tuned KL set-up this does not give a working model. In section 4 we combine the modulus and inflaton sectors by their multiplying superpotentials, or equivalently by adding their Kähler-functions. The modulus corrections to the inflaton potential now are under control, and for a certain range of parameters we get successful inflation. The parameter range for which the standard F-term hybrid inflation predictions apply is determined in section 5. We end with some concluding remarks.

Throughout this article we will work in units with $M_{\rm pl} = 1/\sqrt{8\pi G_N} = 1$.

2. Background

2.1. SUSY F-term hybrid inflation

The superpotential for standard SUSY F-term hybrid inflation is [19, 20]

$$W_{\rm inf} = \lambda\phi(\phi^+\phi^- - v^2)\,. \qquad (1)$$

with ϕ the singlet inflaton field, and $\phi^\pm$ the waterfall fields with charges ±1 under some $U(1)$ symmetry. We can make λ real by an overall phase rotation of the superpotential, whereas the phase of v can be absorbed in the waterfall fields. This is the convention we will use throughout this paper. In particular, in sections 3 and 4 where we combine

inflation with a moduli stabilisation potential, all residual phases reside in the moduli superpotential. The scalar potential is

$$V_{\rm inf} = \lambda^2|\phi|^2\left(|\phi^+|^2 + |\phi^-|^2\right) + \lambda^2\left|\phi^+\phi^- - v^2\right|^2 + V_D\,. \tag{2}$$

Vanishing of the D-term potential enforces $|\phi^+| = |\phi^-|$. Inflation takes place for $|\phi| > v$, during which the waterfall fields sit at the origin $\phi^\pm = 0$. The potential then reduces to a constant energy density

$$V_{\rm inf} = V_* \equiv \lambda^2 v^4\,, \tag{3}$$

which drives inflation. The inflaton potential is flat at tree level, but quantum corrections generate a slope for the inflaton field. The one-loop potential is given by the Coleman-Weinberg formula [21, 22]

$$V_{\rm loop} = \frac{1}{32\pi^2}{\rm Str}M^2\Lambda^2 + \frac{1}{64\pi^2}{\rm Str}M^4\left(\log\frac{M^2}{\Lambda^2} - \frac{3}{2}\right)\,, \tag{4}$$

with the supertrace defined as ${\rm Str}f(M) = f(M_{\rm (boson)}) - f(M_{\rm (fermion)})$, and Λ is the cut-off scale. During inflation SUSY is broken and the masses of the waterfall field and their superpartners are split

$$m_\pm^2 = \lambda^2(|\phi|^2 \pm v^2)\,, \qquad \tilde{m}_\pm^2 = \lambda^2|\phi|^2\,, \tag{5}$$

giving a non-zero contribution to the logarithmic term in $V_{\rm loop}$. Inflation ends when the inflaton drops below the critical value $|\phi| = v$, and one combination of the waterfall fields becomes tachyonic. During the phase transition ending inflation the $U(1)$ symmetry gets broken and cosmic strings form according to the Kibble mechanism [23, 24].

The predictions for the CMB power spectrum and spectral index are

$$P = \frac{V}{150\pi^2\epsilon}\,, \qquad n_s = 1 - \frac{d\ln P(N)}{dN} \approx 1 + 2\eta - 6\epsilon\,, \tag{6}$$

evaluated at $N = N_* \sim 60$, where $N = -\log a$ is the number of e-folds before the end of inflation. The slow-roll parameters are $\epsilon = (1/2)\left(V'/V\right)^2$ and $\eta = V''/V$, with primes denoting differentiation with respect to the canonically normalised real inflaton field φ, which for the above model is $\varphi = \sqrt{2}|\phi|$. The COBE normalisation [25] for the power spectrum is $P \approx 4\times10^{-10}$, and WMAP3 results [26] give $n_s \approx 0.95 \pm 0.02$. We note however that if cosmic strings give a minor contribution to the power spectrum, larger values of the spectral index are favoured [27].

We can get approximate analytical expressions in two limiting cases. For large couplings $\lambda^2 \gtrsim 7.4\times10^{-6}$ inflation takes place for large field values $\varphi \gg v$, and the potential including loop corrections approximates to

$$V_{\rm inf} \approx V_*\left[1 + \frac{\lambda^2}{8\pi^2}\log\frac{\lambda\varphi}{\sqrt{2}\Lambda}\right]\,. \tag{7}$$

It follows that N e-folds before the end of inflation, the inflaton field is $\varphi \approx \lambda\sqrt{N}/(2\pi)$. The prediction for the power spectrum is $P \approx 16N_* v^4/75$, which when normalised to the COBE scale gives $v^2 \approx 5.6\times10^{-6}$. The spectral index is $n_s \approx 1 - 1/N_* \approx 0.98$. In the opposite limit, $\lambda^2 \lesssim 7.4\times10^{-6}$, inflation takes place for inflaton values close

to the critical value $\varphi_* \approx \varphi_{\rm end} \approx \sqrt{2}v$. Fitting the power spectrum to the COBE normalisation now gives $v^2 = 5.6 \times 10^{-6}[\lambda^2/(7.4 \times 10^{-6})]^{1/3}$, and an approximately scale invariant spectrum $n_s \approx 1$.

Cosmic strings can contribute up to about 10% (depending on the angular scale) to the CMB power spectrum [27, 28, 29]. This puts an upper bound on the string tension, and equivalently on the symmetry breaking scale $v^2 < 10^{-5} — 10^{-6}$, which implies $\lambda < 10^{-3} — 10^{-4}$ [30, 31]. However there are ways to avoid cosmic string production, or at least relax the bound [32]. In any case, the precise inflationary predictions and the issue of cosmic strings is not the main point of this paper. Even if ruled out by future data, F-term hybrid inflation still serves as a useful toy model to study the effects of a moduli sector on inflation. In particular it provides an explicit example for which multiplying superpotentials, instead of adding them, helps to keep the moduli corrections under control.

2.2. SUGRA F-term hybrid inflation

Generically when an inflaton model is extended to include supergravity corrections the potential develops a large curvature, resulting in a slow-roll parameter $\eta \sim 1$ that is far too large for inflation [2, 3]. For F-term hybrid inflation with a canonically normalised inflaton field this curvature correction miraculously vanishes [33]. However, when higher order corrections to the the Kähler potential are taken into account, or when a modulus sector is included, this accidental cancellation is destroyed, and the η-problem reappears. It can be solved by introducing a shift symmetry for the inflaton field into the inflationary Kähler potential [6, 12]

$$K_{\rm inf} = -\frac{(\phi - \bar{\phi})^2}{2} + |\phi^+|^2 + |\phi^-|^2 \,. \qquad (8)$$

The canonically normalised inflaton, which is now $\varphi = \sqrt{2}\,{\rm Re}(\phi)$ (rather than $|\phi|$), does not appear explicitly in the Kähler.

However, the SUGRA model with Kähler (8) and superpotential (1) still does not work. The reason is that the mass of the axion field $a = \sqrt{2}\,{\rm Im}(\phi)$ is tachyonic: $m_a^2 = -3\lambda^2 v^4$. This problem is solved if we include an extra no-scale modulus field T in the model. Explicitly, take $K = -3\ln(T + \bar{T}) + K_{\rm inf}$ and

$$W_{\rm inf} = \lambda_0 \phi(\phi^+\phi^- - v_0^2) \,. \qquad (9)$$

The modulus field T can arise in string theory as the breathing mode of compactified extra dimensions; we will discuss it in more detail in the next subsection. In the limit that T is fixed we recover (3) with $v = v_0$, and $\lambda = \lambda_0(2\,{\rm Re}\,T)^{-3/2}$ the rescaled coupling. The mass of the axion field is now positive definite $m_a^2 = 2\lambda^2 v^4(3 + 2\phi^2)$. The masses of the waterfall fields are also altered

$$m_\pm^2 = \lambda^2[\phi^2 + v^4(1+\phi^2) \pm v^2(1+2\phi^2)] \,, \qquad \tilde{m}_\pm^2 = \lambda^2|\phi|^2. \qquad (10)$$

Since $v \ll 1$ the v^4 term is negligbly small. For $\lambda \lesssim 0.5$ we have $\phi^2 \lesssim 1$, and the other correction is also small. The waterfield masses then reduce to the global SUSY results (5), and the model approaches the SUSY limit.

This is all very well, but in the above discussion we have neglected to include a stabilisation mechanism for the modulus T. The full theory must include additional potential terms, which break SUSY and are expected to give corrections to the effective inflaton potential. This is actually part of a wider issue, namely that inflation does not exist in isolation — it is part of a full theory containing other very high energy physics (such a stabilisation mechanisms for moduli fields like T). Given the restrictive form of SUGRA theories, interaction between different sectors is unavoidable (gravity couples to everything). As we will see in later sections, this can be catastrophic for many apparently good theories, and leads to severe restrictions on others. Before discussing the moduli corrections to inflation, we will first review moduli stabilisation in the KKLT and KL set-ups.

2.3. KKLT and KL moduli stabilisation

KKLT devised an explicit method for constructing dS or Minkowksi vacua in string theory [4]. In their set-up all moduli fields are fixed by fluxes [34], except for the volume modulus T which is stabilised by the superpotential

$$\mathcal{W}_{\rm KKLT} = W_0 + A\mathrm{e}^{-aT}\,, \qquad \mathcal{K} = -3\log[T+\bar{T}]\,, \tag{11}$$

where W_0 comes from fluxes, and the non-perturbative exponential term from gaugino condensation or alternatively from instanton effects. For a general SUGRA theory, the F-term potential is

$$\mathcal{V}_F = \mathrm{e}^{\mathcal{K}}\left(\mathcal{K}^{I\bar{J}}D_I\mathcal{W}\bar{D}_{\bar{J}}\bar{\mathcal{W}} - 3|\mathcal{W}|^2\right) \tag{12}$$

with $D_I\mathcal{W} = \mathcal{W}_{,I} + \mathcal{K}_I\mathcal{W}$. The minimum of the above superpotential (11) is SUSY preserving and AdS. However, we require a Minkowski or dS vacuum with a small cosmological constant to desribe our universe. This can be obtained by adding an uplifting term, which then gives a minumum in which SUSY is broken. In the original KKLT paper an anti-D-brane was used for uplifting. Alternatively a D-term can be used [35] although additional meson fields are required to implement this [36]. D-term uplifting has the advantage that the full theory can still be described by SUGRA, whereas the KKLT uplifting term breaks SUSY explicitly. In this paper we assume any lifting term takes the form

$$\mathcal{V}_{\rm lift} \propto \frac{K_T^2}{\mathrm{Re}\, f(T)}\,, \tag{13}$$

where $f(T) \propto T$, or is a constant. This gives the correct form for the KKLT lifting $\mathcal{V}_{\rm lift} \propto (\mathrm{Re}\, T)^{-n}$ with $n = 2, 3$. The D-term will also include the meson fields, although $\mathcal{V}_{\rm lift}$ is qualatively the same (at least for the analysis of this paper).

Alternatively one can introduce an uplifting F-term sector, such as an O'Raifeartaigh [37] or ISS [38] sector. An explicit example of this is the O'KKLT model [13], in which a minimal O'Raifeartaigh sector is added to (11). In this paper we

will implement the theory with a no-scale Kähler. The full moduli stabilisation sector is then

$$\mathcal{K} = -3\ln\left[T + \bar{T} - \frac{K_{O'}}{3}\right], \qquad \mathcal{W} = \mathcal{W}_{\rm KKLT} + \mathcal{W}_{O'} \tag{14}$$

with

$$\mathcal{K}_{O'} = S\bar{S} - \frac{(S\bar{S})^2}{\Lambda_s^2}, \qquad \mathcal{W}_{O'} = -\mu^2 S\,. \tag{15}$$

The O'Raifeartaigh sector breaks SUSY and lifts the AdS vacuum to Minkowski. There is then no need for a separate non-F lifting term in the theory.

The resulting stabilisation potential $\mathcal{V}_{\rm mod} = \mathcal{V}_F + \mathcal{V}_{\rm lift}$ has only one scale $m_T \sim m_{3/2}$. The Minkowski minimum is separated from $T = \infty$ by a barrier of height $\mathcal{V}_{\max} \sim m_T^2$. The barrier needs to be higher than the inflationary scale, otherwise the moduli will roll off to infinity and the internal space will be decompactified, which gives the bound $H_* < m_{3/2}$ on the inflationary scale [5].

KL devised a moduli stabilisation scheme that circumvents the above bound on the Hubble scale during inflation [13]. Instead of the KKLT superpotential they use a modified racetrack superpotential

$$\mathcal{W}_{\rm KL} = W_0 + A\mathrm{e}^{-aT} + B\mathrm{e}^{-bT}\,. \tag{16}$$

The extra parameters in the superpotential allow us to tune $\mathcal{W}_{,T} = \mathcal{W} = 0$, giving a metastable SUSY Minkowski vacuum without the need for a lifting term. As it stands, the model has $m_{3/2} = 0$. This can be avoided by slightly perturbing the Minkowski solution to obtain an AdS minimum $V \sim -m_{3/2}^2 \ll m_T^2$, which is then uplifted to a SUSY breaking Minkowski vacuum. Uplifting can be done with a small KKLT lifting term, or alternatively by adding an uplifting F-term sector (15), as was used in section 3 of [13]. If the SUSY-breaking scale is small, we have $T\mathcal{W}_{,T} \sim \mathcal{W} \sim m_{3/2}T^{3/2}$ and the gravitino mass is far smaller than the modulus mass scale, which is typically set by W_0 in the superpotential. It is then possible to have $m_{3/2}^2 \ll H_*^2 \ll \mathcal{V}_{\max} \sim m_T^2$, which opens the possibility of having inflation with fixed moduli but small soft corrections to the inflaton potential. Note that such a scenario cannot be implemented with an uplifting D-term. In this case gauge symmetry implies that the Minkowski solution $\mathcal{W}_{,T} = \mathcal{W} = 0$ is obtained along a flat direction in the meson-modulus field space. As a result, after perturbing the solution and uplifting to Minkowski, only one modulus mass eigenstate is large. The other is only $\mathrm{O}(m_{3/2})$, and so the barrier height along the previously flat direction is also small $\mathcal{V}_{\max} \sim m_{3/2}^2$, even when the modulus mass is large $m_T \gg m_{3/2}$.

The above model (14) uses a slightly different $\mathcal{K}$ to [13], although it has similar properties. We have chosen the above Kähler to simplify the analytical expressions. But we want to emphasise that the exact way the modulus potential and the O'Raifeartaigh section are combined does not significantly affect inflation. For that matter, the uplifting sector does not have to be O'Raifeartaigh either, but can be some other F-term SUSY breaking sector such as the ISS model. The differences in the resulting potential will

be of order $\mathrm{O}(m_{3/2}^2)$, and as long as $m_{3/2} \ll H_*$ such differences are irrelevant during inflation. As we will see in section 4, whether the uplifting is F-term or not can make a major difference. For the case where the modulus and inflaton sector are combined by adding their respective Kähler functions it is the difference between a viable model and no model at all.

3. Combining inflation and moduli stabilisation by addition

The usual way to combine the models of the previous sections is to add the respective superpotentials $W = \mathcal{W} + W_{\rm inf}$. Here $\mathcal{W}$ is the modulus superpotential, either KKLT (11) or KL (16), possibily including an F-term O'Raifeartaigh lifting sector. For the Kähler potential we consider the simplest possibility

$$K = -3\ln\left[X\right] + K_{\rm inf}\,, \tag{17}$$

with

$$X = T + \bar{T} - \frac{\mathcal{K}_{O'}}{3}\,. \tag{18}$$

If uplifting is achieved via an anti-D-brane or D-term, $\mathcal{W}_{O'}$ and $\mathcal{K}_{O'}$ are simply set to zero. To verify that the qualitative results are independent of the exact form of the Kähler, we also consider the more general expression

$$K = -3\ln\left[X - X^{\alpha}\frac{K_{\rm inf}}{3}\right]\,. \tag{19}$$

For $\alpha = 0$ this gives a fully no-scale Kähler potential: $K_a K^{a\bar{b}} K_{\bar{b}} = 3$ with a, b running over both moduli and inflaton fields.

Slow-roll inflation with a scale invariant spectrum of perturbations requires $\epsilon, \eta \ll 1$. Hence we have to make sure the moduli induced corrections to the slope and curvature of the inflaton potential are sufficiently small. The corrections to the masses of the waterfall and axion fields must also be small. If the mass corrections to the waterfall fields are too large and positive definite, they prevent $\phi^{\pm}$ becoming tachyonic, and there is no exit from inflation. Alternatively, if the corrections are large and tachyonic the system ends up in the wrong vacuum. Furthermore, the axion mass has to be positive definite during inflation, which is not automatic. For the moment we work in the approximation that the moduli are fixed at the minimum $T = T_0$ during inflation. At the end of this section we will drop this assumption, and analyse its implications.

For either choice of Kähler we find there are corrections to the slope of the inflationary potential [14]. For (17), the full F-term potential for the combined theory is

$$V_F = e^{K_{\rm inf}}\mathcal{V}_F + \mathcal{V}_{\rm lift} + e^K|\partial_i W_{\rm inf} + K_i(W_{\rm inf} + \mathcal{W})|^2 + V_{\rm mix}\,, \tag{20}$$

which is roughly the sum of the potential for the inflation and moduli sectors (with some rescaling), and the additional mixing terms

$$V_{\rm mix} = 2e^K\,\mathrm{Re}[(\mathcal{K}^{I\bar{J}} D_I\mathcal{W}\mathcal{K}_{\bar{J}} - 3\mathcal{W})\bar{W}_{\rm inf})] + e^K(\mathcal{K}^{I\bar{J}}\mathcal{K}_I\mathcal{K}_{\bar{J}} - 3)|W_{\rm inf}|^2\,. \tag{21}$$

The index i runs over the inflation sector fields, while I, J run over the moduli sector fields. During inflation all $K_i = 0$, and the SUGRA $K_i W$ corrections to SUSY inflation vanish. Furthermore, for a no-scale moduli Kähler (17) the second term of $V_{\rm mix}$ is identically zero. The Kähler potential (19) gives rise to similar mixing terms.

Much of the moduli interaction effectively re-scales the inflationary parameters, and so it is convenient to introduce

$$\lambda = \frac{\lambda_0}{X^{3\alpha/2}}\,, \qquad v^2 = \frac{v_0^2}{X^{1-\alpha}}\,, \qquad V_* = 3H_*^2 = \lambda^2 v^4\,, \qquad \varphi = \sqrt{2X^{\alpha-1}}\,\mathrm{Re}\,\phi\,. \tag{22}$$

These apply to the general Kähler (19), and also to (17) if α is set to 1. In both cases the inflationary potential reduces to

$$V_{\rm inf} = V_* + \mathcal{V}_{\rm mod} + \frac{\sqrt{2}\,\mathrm{Re}(\mathcal{W}_{,T})}{\sqrt{X}}\lambda v^2 \varphi \tag{23}$$

with φ the canonically normalised inflaton. The inflaton independent modulus potential is $\mathcal{V}_{\rm mod}(T) = \mathcal{V}_F + \mathcal{V}_{\rm lift}$. We see that a nearly flat inflaton potential, with $\epsilon \ll 1$, requires $V_{\rm mix} \propto \mathrm{Re}\,\mathcal{W}_{,T}$ to be small. This can be achieved either be making $|\mathcal{W}_{,T}|$ small (which is the case for the two-scale KL-style stabilisation), or by having $\mathcal{W}_{,T}$ imaginary, i.e. having a phase difference between the inflation and moduli superpotentials.

We also need to check that the corrections to the masses of the waterfall fields do not radically change the ending of inflation, and that the axion $a = \sqrt{2}\,\mathrm{Im}\,\phi$ remains stable. We introduce the mass scales

$$m = \frac{\mathcal{W}}{X^{3/2}}\,, \qquad m' = \frac{\mathcal{W}_{,T}}{\sqrt{X}}\,, \qquad \mathcal{M} = \frac{\sqrt{X}\mathcal{W}_{,TT}}{3}\,. \tag{24}$$

Up to small $\mathrm{O}(\mathrm{e}^{K_{\rm inf}})$ corrections $|m| \approx m_{3/2}$ is the gravitino mass after inflation, and in a KL-style scheme $|\mathcal{M}| \approx m_T$ the modulus mass. For KKLT we still have $|\mathcal{M}| \sim m_T$. For the Kähler (17) with canonically normalised inflaton sector fields the masses of the axion and waterfall fields are

$$m_a^2 = 2\lambda^2 v^4(3 + 2\phi^2) + 2\mathcal{V}_F + 4|m|^2 - 4\,\mathrm{Re}[2m - m']\lambda v^2 \phi\,, \tag{25}$$

$$m_\pm^2 = \lambda^2\phi^2 \pm \lambda^2 v^2 \left| 1 - \frac{2m - m'}{\lambda v^2}\phi + 2\phi^2 \right| + \mathcal{V}_F + |m|^2 + \lambda^2 v^4(1+\phi^2) + 2\lambda v^2\,\mathrm{Re}[m' - m]\phi\,. \tag{26}$$

For a one-scale KKLT-like moduli sector $m, m' \sim m_{3/2} \sim m_T$. The requirement that the moduli remain fixed during inflation, i.e. $H_*^2 < \mathcal{V}_{\rm max} \sim m_T^2$, implies that the $\mathrm{O}(m, m')$ moduli sector corrections to $m_\pm^2$ dominate, preventing a gracefull exit from inflation. A further problem for models which use a D-brane or D-term lifting term $\mathcal{V}_{\rm lift}$ is that the axion and waterfall masses recieve large tachyonic contributions from the moduli sector F-term potential $\propto \mathcal{V}_F \sim -3m_{3/2}^2$. For F-term lifting $\mathcal{V}_F = 0$ in the Minkowski vacuum after inflation, and so the contribution of $\mathcal{V}_F$ during inflation is small.

In principle, all these problems can be avoided with sufficient fine-tuning, although the single mass scale superpotential (11) does not contain enough parameters. Hence we must switch to a two-scale KL moduli stabilisation scheme, which is tuned so that $\mathcal{W} = \mathcal{W}_{,T} \approx 0$ and thus $m, m', \mathcal{V}_F \approx 0$. The moduli corrections to the waterfall (26)

and axion (25) field masses, as well as to the inflaton potential (23), are then negligibly small during inflation.

So it appears that the potential can be kept flat and the mass corrections small in a KL-style set-up. But as we will now show this is not the final picture. In the above analysis we assumed T was fixed at the minimum of $\mathcal{V}_{\rm mod}$. However, no field is truly fixed at a constant value during inflation, and in particular the modulus minimum will shift slightly during inflation. Taking the dynamics of the modulus into account, we will now show that it produces siginificant curvature corrections to the potential, and consequently gives too large a value for η [14]. To do so we Taylor expand the potential (23) in $\delta T = T - T_0$, with as before T_0 the modulus value that minimises the *post*-inflationary potential:

$$V_{\rm inf} = V_*(T_0) + \mathcal{V}_{\rm mod}(T_0) + \frac{2\,{\rm Re}\,\mathcal{W}_T(T_0)}{X^2}\lambda_0 v_0^2\phi + \delta V_{\rm inf} + {\rm O}\big(|\delta T|^3, \lambda_0 v_0^2\phi|\delta T|, V_*|\delta T|\big) \quad (27)$$

where

$$\begin{aligned}\delta V_{\rm inf} = {}& \mathcal{V}_{{\rm mod},T\bar{T}}\,\delta T\overline{\delta T} + {\rm Re}[\mathcal{V}_{{\rm mod},TT}\,\delta T^2] \\ &+ 2\,[X\,{\rm Re}(\mathcal{W}_{,TT}\,\delta T) - 4\,{\rm Re}(\mathcal{W}_{,T})\,{\rm Re}(\delta T)]\,\frac{\lambda_0 v_0^2}{X^3}\phi \end{aligned} \quad (28)$$

gives the leading order corrections to $V_{\rm inf}$ from the variation of T. Now for KL $|\mathcal{M}| \gg |m|, |m'|$, hence this reduces to

$$\delta V_{\rm inf} \approx 3\frac{|\mathcal{M}|^2}{X^2}|\delta T|^2 + \frac{3\sqrt{2}\lambda v^2\varphi}{X}\,{\rm Re}[\mathcal{M}\,\delta T]\,. \quad (29)$$

Minimising with respect to δT we find

$$\frac{\delta T}{X} \approx -\frac{\lambda v^2\varphi}{\sqrt{2}\mathcal{M}} \quad (30)$$

which is small (as expected). However when this is substituted back into the above potential, it produces a large negative inflaton mass

$$\delta V_{\rm inf} \approx -\frac{3}{2}V_*\varphi^2\,. \quad (31)$$

The η-problem rears its head again: $\eta = V_{,\varphi\varphi}/V \approx -3$. For KL without the SUSY breaking O'Raifeartaigh sector the above expressions are exact, while an uplifting sector — O'Raifeartaigh or otherwise — gives rise to small ${\rm O}(m_{3/2}^2)$ corrections (both due to the above δT expression, as well as the displacement of e.g. the O'Raifeartaigh field δS). The large slow-roll parameter rules out F-term hybrid inflation with KL moduli stabilisation. The reason for the large corrections, even in the fine-tuned KL set-up is that although $\mathcal{W} \sim \mathcal{W}_{,T} \approx 0$ are small, $|\mathcal{W}_{,TT}|^2 = 3X\mathcal{V}_{{\rm mod},T\bar{T}} + {\rm O}(\mathcal{M}m_{3/2})$ is not. In the Minkowski vacuum after inflation the potential is fine-tuned so that $m_{3/2}^2 \ll m_T^2$, but during inflation, due to the small displacement of the modulus field, this tuning is disrupted, and corrections are large.

For the more general Kähler (19) the inflaton potential is still given by (23). The waterfall masses take the form

$$m_\pm^2 = \frac{\lambda^2\varphi^2}{2} \pm \lambda^2 v^2\left|1 + \frac{(1+2\alpha)m' - 6\alpha m}{3\sqrt{2}\lambda v^2}\varphi + \alpha\varphi^2\right| + \frac{2+\alpha}{3}\mathcal{V}_F + \frac{2(1-\alpha)}{3}\mathcal{V}_{\rm lift}$$

$$+\alpha|m|^2+\frac{\sqrt{2}\lambda v^2}{3}\,\mathrm{Re}[(2+\alpha)m'-3\alpha m]\varphi+\lambda^2 v^4\left(\frac{2}{3}+\frac{\alpha\varphi^2}{2}\right). \qquad (32)$$

In general, the model will have all the same problems as that arising from the simpler Kähler (17), and one-scale KKLT-style moduli stabilisation superpotentials are ruled out. It is interesting to note that for a no-scale $\alpha = 0$ model most of the corrections to $m_\pm$ cancel (compare with the D-term inflation model proposed in [16]). In particular, all the m, $\mathcal{V}_F$ and $\mathcal{V}_{\rm lift}$ corrections disappear. It would seem that we then only need to impose the single fine-tuning $m' \approx 0$, to obtain a viable inflation model. Unfortunately the KKLT superpotential (11) does not have enough freedom to do this, and viable inflation is not obtained. Furthermore, the above discussion does not take into account the varation of T during inflation. The above analysis of δT also applies for the more general Kähler (19), and so it too is ruled out.

To conclude, F-term hybrid inflation does not work for either KKLT- or KL-style moduli stabilisation, no matter what the form the Kähler takes, at least if we combine the inflation and modulus sector by adding superpotentials. In fact, if more exponential terms are added to the moduli stabilisation superpotential, its first three derivatives are appropriately tuned, and the Kähler is carefully choosen, the moduli dynamics could be different to those used to get (31). A viable model of inflation could concievably be constructed, although it is hard to justify all the fine-tuning. Furthermore, there is no guarantee that additional problems will not arise as a result of this tuning. We will not consider such as set-up here, and will instead turn to a much more elegant solution.

4. Combining inflation and moduli stabilisation by multiplication

The inflaton and modulus sectors can also be combined by multiplying their superpotentials. Although due to its unfamiliarity this seems strange at first, we argue that from a supergravity point of view it is a rather natural thing to do. Multiplying superpotentials greatly reduces the mixing between sectors [1, 18]. Indeed, as we will discuss in this section F-term hybrid inflation combined in this way with KL or even a KKLT moduli sector gives a viable inflation model.

The supergravity formulation in terms of K and W is redundant, as a Kähler transformation leaves the theory invariant. Instead the theory can be formulated in terms of single Kähler invariant function $G = K + \ln|W|^2$, which is known as the Kähler function. The kinetic terms and F-term potential are then given in terms of G only. This suggests that the Kähler function is a more “fundamental” or “natural” quantity to consider. Hence when combining sectors, it may be argued that one should add their respective Kähler functions, which corresponds to adding Kähler potentials and multiplying superpotentials.

For the combined theory we then take $G = G_{\rm mod} + G_{\rm inf}$. The reduced inflaton-moduli interactions are a result of the following property. Consider a SUSY critical point $T = T_0$ of the modulus sector $\partial_T G_{\rm mod}(T_0) = 0$, which corresponds to a SUSY extremum of the moduli potential. It can easily be shown that this is then a SUSY critical point

of the full theory as well $\partial_T G(T_0) = 0$ [1, 18]. This is exactly what we want, as it implies that the modulus minimum is not shifted during inflation. The δT corrections to the potential, which were fatal when adding superpotentials, are then absent. Of course, with SUSY broken in the modulus sector the minimum of the modulus potential is not exactly in a critical point. But in the KL-like set-up the deviations away from the SUSY critical point are small, of the order of the small gravitino mass. Consequently we expect the modulus field to be nearly constant during inflation, and the corresponding correction to the potential to be suppressed by the smallness of the gravitino mass. As we will see, this is indeed the case.

One disadvantage of the Kähler function formulation of SUGRA is that it is ill defined whenever $W = 0$. This presents a problem for F-term hybrid inflation, as the inflationary superpotential (1) is zero after inflation. To solve this problem we "correct" the superpotential by adding a constant

$$W_{\rm inf} = \lambda_0 \phi(\phi^+\phi^- - v_0^2) - C\,. \tag{33}$$

Here we will assume that C is real and positive, although generalisation of the analysis to include a phase is straightforward. The constant C is of course irrelevant in the IR global SUSY limit, whereas in the UV regime it makes the model well behaved. Similarly, for the modulus potential we cannot take the supersymmetric KL limit, a finite amount of SUSY breaking (explicitly provided in (16) by an O'Raifeartaigh sector) is required. The effective superpotential of the model with the modulus included is now

$$W = \mathcal{W}\, W_{\rm inf}\,. \tag{34}$$

For the Kähler potential we still use (17) with canonically normalised inflaton fields. To test the dependence of the results on the exact form of the Kähler we also give the results for the general expression (19).

For the minimal Kähler (17) the potential that follows from (33),(34) is

$$V = e^{K_{\rm inf}}|W_{\rm inf}|^2\mathcal{V}_F + e^{\mathcal{K}}|\mathcal{W}|^2 e^{K_{\rm inf}}\left|\partial_i W_{\rm inf} + K_i W_{\rm inf}\right|^2 + \mathcal{V}_{\rm lift}\,. \tag{35}$$

As advertised, the mixing between the inflaton and modulus sector is drastically reduced compared to the case of adding superpotentials (21). The main effect is just a re-scaling of the potential. We define the re-scaled quantities

$$\lambda = \frac{\lambda_0|\mathcal{W}|}{X^{3\alpha/2}}\,, \qquad v^2 = \frac{v_0^2}{X^{1-\alpha}}\,, \qquad \mathcal{V}_{\rm mod} = C^2\mathcal{V}_F + \mathcal{V}_{\rm lift}\,, \qquad \varphi = \sqrt{2X^{\alpha-1}}\,{\rm Re}\,\phi\,. \tag{36}$$

$V_* = 3H_*^2 = \lambda^2 v^4$ is then the rescaled inflationary potential driving inflation, while $\mathcal{V}_{\rm mod}$ is the full rescaled modulus stabilsation potential after inflation. The field φ is the real, canonically normalised, inflaton field. As before, the expressions for (17) correspond to $\alpha = 1$. We also define the mass scales

$$m = \frac{C|\mathcal{W}|}{X^{3/2}}\,, \qquad \mathcal{M} = \frac{C\sqrt{X}|\mathcal{W}_{,TT}|}{3}\,, \tag{37}$$

which can respectively be thought of as the gravitino and moduli mass in the vacuum, after inflation. With these definitions the potential during inflation for both (17) and (19) reduces to

$$V_{\rm inf} = V_* + V_{\rm mod}\left(1+\frac{\lambda v^2\varphi}{\sqrt{2}m}\right)^2 - \frac{\lambda v^2\varphi}{\sqrt{2}m}\left(2+\frac{\lambda v^2\varphi}{\sqrt{2}m}\right)\mathcal{V}_{\rm lift}\,. \qquad (38)$$

We see that if a seperate lifting term is present (either an anti-D-brane or a D-term), its potential $\mathcal{V}_{\rm lift} \sim m_{3/2}^2$ gives a large negative contribution to η. This holds for both the KKLT and KL superpotential, and so all our moduli stabilisation scenarios with non-F lifting terms are incompatible with F-term hybrid inflation. In the remainder of this section will thus focus on the case of F-term lifting with $\mathcal{V}_{\rm lift} = 0$.

In the limit that the modulus remains fixed during inflation $\mathcal{V}_{\rm mod} = 0$ for F-term lifting, and there are no corrections to the inflaton potential at all. This is in sharp contrast to the potential obtained when adding superpotentials (23). Although the modulus is not truly fixed during inflation, we will see below that the corrections to this assumption are small.

In multiplying the superpotentials, our intention was to reduce the effect of the moduli sector on inflation. We see from (38) that a beneficial side effect of this is that the inflaton enhances the moduli stabilisation. In particular the barrier height for the moduli stabilisation potential is now

$$V_{\rm max} \sim \mathcal{M}^2\left(1+\frac{\sqrt{3}H_*\varphi}{\sqrt{2}m}\right)^2\,. \qquad (39)$$

Hence we expect the moduli to remain near their minimum during inflation if $\mathcal{M} \gg H_*$ (as is usually assumed), or if $(\mathcal{M}/m)\varphi \gg 1$. Since $\varphi > \varphi_{\rm end} \sim v$, the moduli should be stable thoughout inflation if either

$$(a)\quad \mathcal{M} \gg H_* \qquad \text{or} \qquad (b)\quad \mathcal{M} \gg \frac{m}{v} \gtrsim 4\times 10^2\, m\,. \qquad (40)$$

Significantly, the second possibility does not depend on the Hubble constant during inflation, and so having $H_* > \mathcal{M}$ is not a problem. The $H_* < \mathcal{M}$ bound was a major motivation for the KL scenario, and its removal suggests that a two-scale, KL-style moduli sector is no longer needed. However, while the bound (40b) is easily satisfied for KL, it cannot be satisfied by KKLT. Hence it seems that a two-scale KL-like moduli sector is needed after all, although not necessarily for the reasons that were originally envisaged.

For the simplest Kähler (17) the waterfall field masses are

$$m_\pm^2 = \lambda^2\phi^2 \pm \lambda^2 v^2\left(1+\frac{2m}{\lambda v^2}\phi + 2\phi^2\right) + (m+\lambda v^2\phi)^2 + \lambda^2 v^4\,. \qquad (41)$$

In the limit

$$m \approx m_{3/2} \ll \frac{\lambda v^2}{\varphi} \qquad (42)$$

the moduli corrections are subdominant, and inflation ends as in usual hybrid inflation. From the COBE normalisation it follows that $v^2 \ll 1$ and all v^2 corrections can be

neglected as well. For a KKLT-style superpotential (11) with $m \sim \mathcal{M}$, it is difficult to satisfy both of the above bounds (40), (42) simultaneously, and most vlaues of $\mathcal{M}$ are ruled out. For smaller values of λ (for which $\varphi_* \ll 1$) there is a small window of parameter space $H_* \ll \mathcal{M} \ll H_*/\varphi_*$ where inflation will be viable. For a two-scale KL-style scenario there is more room to satisfy the bounds (40), (42), but at the cost of fine-tuning the potential.

For the more general Kähler (19) the waterfall masses are instead

$$m_\pm^2 = \frac{\lambda^2\varphi^2}{2} \pm \lambda^2 v^2 \left| 1 + \left[\alpha + (1-\alpha)\frac{X\mathcal{W}_{,T}}{3\mathcal{W}}\right]\left[\varphi + \frac{\sqrt{2}m}{\lambda v^2}\right]\varphi \right| + \alpha\left(m + \frac{\lambda v^2 \varphi}{\sqrt{2}}\right)^2 + \frac{2\lambda^2 v^4}{3}\,, \tag{43}$$

For $\alpha \neq 1$ there are additional corrections to the watefall fields proportional to $\mathcal{W}_{,T}$. These are expected to be of the same size as the other corrections. Hence KKLT-style models are again mostly ruled out, except for a small range of $\mathcal{M}$.

We now turn to the behaviour of the moduli fields during inflation. We saw above how a lower bound on $\mathcal{M}$ arises from the requirement that $V_{\max} \gg V_*$. In fact, a stronger bound on $\mathcal{M}$ comes from the inflationary corrections to the moduli sector masses. The respective masses of the real and imaginary parts of T, and their fermionic superpartners are

$$m_{\mathrm{Re}\,T}^2 \approx \tilde{m}_T^2 + \frac{\mathcal{M}}{m}V_*\,, \qquad m_{\mathrm{Im}\,T}^2 \approx \tilde{m}_T^2 - \frac{\mathcal{M}}{m}V_*\,, \qquad \tilde{m}_T^2 \approx \mathcal{M}^2\left(1 + \frac{\lambda v^2\varphi}{\sqrt{2}m}\right)^2 \tag{44}$$

up to $\mathrm{O}(m)$ corrections. To get the above expressions we have used that $|\mathcal{W}_{,TT}|^2 = 3X\mathcal{V}_{\mathrm{mod},T\bar{T}} + \mathrm{O}(\mathcal{M}m)$ in the KL set-up; it should also be remembered that the rescaled coupling λ is modulus dependent. We have assumed, for simplicity, that $\mathcal{W}$ and its derivatives all have the same phase. The masses (44) for KKLT will have different coefficents, but will be qualitatively similar. Requiring that $\mathrm{Im}\,T$ is not tachyonic implies either

$$(a) \quad \mathcal{M} \gtrsim \frac{H_*^2}{m} \qquad \text{or} \qquad (b) \quad \mathcal{M} \gtrsim \frac{m}{v^2} \gtrsim 2\times 10^5\, m\,. \tag{45}$$

For large enough $\mathcal{M}$, (a) is satisfied by KL- and KKLT-style moduli sectors, and can in both cases be combined with (42). The other range (b) is easily satisfied for KL, but not for KKLT.

Finally, we need to check that taking the modulus fixed during inflation, as assumed above, is a good approximation. As we saw in section 3, the modulus dynamics destroys inflation even for the fine-tuned KL set-up when the modulus and inflation superpotentials are added. For a model with multiplied superpotentials, this problem is avoided. We will assume that $\mathcal{W}$ and all its dervatives have the same phases. Expanding, much as before, around the minumum of $\mathcal{V}_{\mathrm{mod}}$, we take $T = T_0 + \delta T_R + \mathrm{i}\delta T_I$. Minimising the resulting potential, we find $\delta T_I = 0$ and

$$\frac{\delta T_R}{X} \approx -\frac{V_*}{3m_{\mathrm{Re}\,T}^2}\left(X\frac{D_T\mathcal{W}}{\mathcal{W}} + 1 - \alpha\right) \tag{46}$$

giving

$$-\frac{\delta V_{\rm inf}}{V_*} \approx \frac{V_*}{3m^2_{{\rm Re}\,T}} \left(\frac{D_T\mathcal{W}}{\mathcal{W}} + \frac{1-\alpha}{X}\right)^2 \lesssim \min\left(\frac{H_*^2}{\mathcal{M}^2}, \frac{m^2}{\mathcal{M}^2\varphi^2}, \frac{m}{\mathcal{M}}\right) . \quad (47)$$

This is just a small correction to the inflationary potential (38) provided that either $\mathcal{M} \gg H_*$, or $\mathcal{M} \gg m$. At least one of these conditions is satisfied if we require that T is not tachyonic during inflation (45).

To summarise, combining the two bounds (42) and (45) gives

$$\sqrt{\mathcal{M}_T m_{3/2}} \gg H_* \gg m_{3/2}\varphi_* \,, \quad (48)$$

or alternatively

$$m_{3/2} \ll \frac{H_*}{\varphi_*} \,,\, \mathcal{M}_T v^2 \,, \quad (49)$$

where $\mathcal{M}_T \approx \mathcal{M}$ is the mass of T after inflation. Either of the above bounds can be satisfied by a KL-style scenario without additional fine-tuning. KKLT-style models can also satisfy bound (48) and give a viable model of inflation for a limited range of $\mathcal{M}$. These conclusions also apply for the more generic, α-dependent Kähler (19). In both KKLT and KL moduli stabilisation potentials, if either of the above bounds is satisfied, then the modulus does not vary significantly during inflation. Hence with only a moderate degree of tuning, inflation can be successfully combined with a modulus sector when their respective superpotentials are multiplied.

5. Inflationary predictions

Having investigated the effects of the moduli stabilisation sector on the tree level inflaton potential, we will now determine the moduli corrections to the one-loop potential. The inflaton slope and curvature, which determine the power spectrum and the spectral index, are dominated by the one-loop contribution. This is given explicitly by the Coleman-Weinberg formula (4). $V_{\rm loop}$ receives contributions from the non-degenerate boson and fermion pairs, which in our model are not only the waterfall fields, but also the modulus field T (we will ignore any other fields for simplicity). Since the masses are φ-dependent, their contribution to the loop potential will generate a non-trivial potential for the inflaton field. In the limit that the slope and curvature of the inflaton potential is dominated by the waterfall field contribution to the loop potential, the inflationary predictions are the same as for the global SUSY model discussed in subsection 2.1. We will then have a working model of inflation. In this section we will determine the corresponding parameter space. More precise bounds could be obtained by comparison with the WMAP data, although the results will be sensitive to the details of the moduli superpotential. Here, we will content ourselves with order of magnitude bounds. Like the conclusions of the previous section, our results will apply to the simple Kähler (17), and to the more generic one (19) for any choice of α.

We start by calculating the loop potential. In the limit that the gravitino mass is small and the bound (42) is satisfied, the expressions for the waterfall masses approach

the SUGRA results (10). If we further restrict to the regime $\varphi < 1$ or $\lambda \lesssim 0.5$, where the results are manifestly cut-off independent, we retrieve the global SUSY results (5). The loop potential due to the waterfall fields is given by the familiar expression [20]

$$V_{\rm loop}^{(\phi)} = \frac{\lambda^2 V_*}{32\pi^2}\left[2\ln\left(\frac{\lambda^2 v^2 x^2}{\Lambda^2}\right) + (x^2+1)^2\ln(1+x^{-2}) + (x^2-1)^2\ln(1-x^{-2}) - 3\right] \quad (50)$$

with $x^2 = \varphi^2/(2v^2)$. Inflation takes place for $x > 1$ and ends as $x \to 1$ with the tachyonic instability. Using (44) the modulus contribution to the loop potential is

$$V_{\rm loop}^{(T)} = \frac{V_*^2\mathcal{M}^2}{64\pi^2 m^2}\left[2\ln\left(\frac{V_*\mathcal{M}z^2}{\Lambda^2 m}\right) + (z^2+1)^2\ln(1+z^{-2}) + (z^2-1)^2\ln(1-z^{-2}) - 3\right] \quad (51)$$

with

$$z^2 = \frac{\tilde{m}_T^2 m}{V_*\mathcal{M}} = \frac{\mathcal{M}}{m}\left(\frac{m}{\lambda v^2} + \frac{\varphi}{\sqrt{2}}\right)^2. \quad (52)$$

The loop potential gives a negligible contribution to the total energy density during inflation V_*, but it is the dominant contribution to the slow-roll parameters ϵ and η. Hence to see whether it is the waterfall or the modulus contribution to the potential which dominates the inflationary dynamics, we have to compare their derivatives. In addition we need to satisfy the upper bound on m (42), so that neglecting $\mathrm{O}(m)$ terms is a good approximation. Requiring that the axion is non-tachyonic during inflation gives a further, lower bound on the modulus mass scale $\mathcal{M}$ (45). Finally, we note that both KKLT and KL moduli stabilisation potentials have $m \lesssim \mathcal{M}$, which restricts the allowed parameter space. If the above constraints are satisfied, then the modulus automatically remains fixed during inflation, and its dynamics do not produce further constraints.

We expect to retrieve standard hybrid inflation results in the limit that the mass splitting between the modulus field and its superpartners is small, as this sets the overall scale of the modulus loop potential. In this limit $z^2 \gg 1$. The φ-dependence only enters $V_{\rm loop}^{(T)}$ via $\tilde{m}_T^2$, and we find it convenient to write

$$\tilde{m}_T = \mathcal{M}(1+\delta_m)\,, \quad \text{with} \quad \delta_m = \frac{\lambda v^2 \varphi}{\sqrt{2}m}. \quad (53)$$

The modulus loop effects are suppressed in the limit $\delta_m \to 0$. As it turns out the $\delta_m \to 0$ limit can be relaxed, and it will be sufficient to consider the loop potential in the regime $z^2 \gg 1$ in order to determine the allowed parameter space. The modulus contribution in the large z-limit is

$$\left(V_{\rm loop}^{(T)}\right)'_* \approx \frac{\lambda^5 v^{10}\mathcal{M}^2}{16\sqrt{2}\pi^2 m^3}\frac{1}{(1+\delta_m)}. \quad (54)$$

This is to be compared with the equivalent expression for the waterfall field potential.

5.1. Large coupling, $\lambda^2 \gtrsim 10^{-5}$

In the large coupling regime, $\lambda^2 > 7.4\times10^{-6}$, we can approximate (50) by the large x result (7) and

$$\lim_{x\gg1}\left(V_{\rm loop}^{(\phi)}\right)'_* \approx \frac{\lambda^3 v^4}{4\pi\sqrt{N_*}}, \quad (55)$$

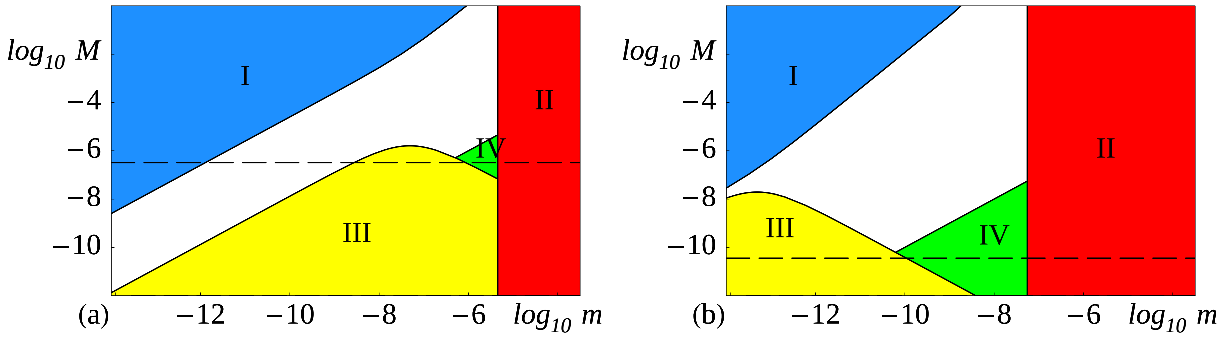

Figure 1. Parameter space in $\{\log_{10}(m), \log_{10}(\mathcal{M})\}$ for (a) $\lambda = 0.1$ and (b) $\lambda = 10^{-4}$. In the white region the model reduces to SUSY hybrid inflation. Regions I-IV are excluded, because I: the modulus mass dominates the 1-loop potential, II: the gravitino mass is too large, III: the modulus is tachyonic during inflation, and IV: the modulus potential property $m \lesssim \mathcal{M}$ is not satisfied. The dashed lines correspond to $H_* = \mathcal{M}$

where we used $\varphi_* \approx \lambda\sqrt{N_*}/(2\pi)$. This dominates over (54) for

$$\mathcal{M}^2 < \frac{4\sqrt{2}\pi m^3(1+\delta_m)}{\sqrt{N_*}\lambda^2 v^6} \approx \begin{cases} 1.3 \times 10^{16}\, m^3 \lambda^{-2}\,, & \delta_m \ll 1 \\ 6.4 \times 10^{10}\, m^2\,, & \delta_m \gg 1 \end{cases} \tag{56}$$

where we used $v^2 \approx 6 \times 10^{-6}$ and $N_* = 60$. Small $m < 4.8 \times 10^{-6}\lambda^2$ corresponds to the large $\delta_m > 1$ regime. This should be combined with the axion mass bound (45) which translates to

$$\mathcal{M} > \frac{\lambda^2 v^4}{m(1+\delta_m)^2} \approx \begin{cases} 3.1 \times 10^{-11}\, m^{-1}\lambda^2\,, & \delta_m \ll 1 \\ 1.3\, m\lambda^{-2}\,, & \delta_m \gg 1 \end{cases} \tag{57}$$

and the bound from modulus corrections to $m_\pm$ (42), which gives $m < 1.4 \times 10^{-5}$.

The parameter space in the $\{\log_{10}(m), \log_{10}(\mathcal{M})\}$-plane is shown for $\lambda = 0.1$ in figure 1a. In the white region the inflationary results approach those of the global SUSY model discussed in section 2.1. Hence, there is a region of parameter space for which multiplying superpotentials gives a viable model of F-term hybrid inflation. This is in sharp contrast to a combined model in which the superpotentials are summed: as we saw in section 3, inflation fails in this case.

In all of parameter space $z^2 \gg 1$, and our analytic results are valid. In region I the loop potential is dominated by the modulus contribution (56); when this becomes too large inflation is ruined. In region II the bound (42) on the gravitino mass is violated, and moduli corrections are too large for successful inflation. Region III is excluded as it gives a tachyonic axion (45). Except for very near the border with region III the η-parameter is dominated by the waterfall field contribution to the loop potential. Finally, region IV bounds $m \lesssim \mathcal{M}$ which is a property of both KKLT and KL-style moduli sectors. Viable, KKLT-style models correspond to the upper-left edge of region IV. Since this class of models has only one mass scale $\mathcal{M} \sim m$, it corresponds to a line in the plotted, two-dimensional parameter space. The fact that $\varphi_* < 1$ during inflation

allows (42) and (45) to be realised simultaneously for a limited range of $\mathcal{M}$ (which increases in size as coupling λ is reduced). The two-scale KL model works throughout the white region of parameter space in the plot.

In the $\delta_m \gg 1$ regime the effective modulus mass is enhanced during inflation compared to its vacuum value $\mathcal{M}$, as can be seen from (39). This allows for the possibility of having $m < \mathcal{M} < H_*$, yet with the modulus fixed during inflation. For $\lambda = 0.1$ the inflationary scale is $H_* \approx 10^{-6}$. The dashed lines in figure 1 correspond to $\mathcal{M} = H_*$; we see that indeed $\mathcal{M} < H_*$ is realised in large part of parameter space, contrary to naive expectations.

5.2. Small coupling, $\lambda^2 \lesssim 10^{-5}$

We can apply the same analysis for the small coupling regime $\lambda^2 < 7.4 \times 10^{-6}$. In this case $\varphi_* \approx \sqrt{2}v$ and $v^2 = 5.6 \times 10^{-6}[\lambda^2/(7.4 \times 10^{-6})]^{1/3}$. In the small $x \to 1$ limit the slope of the waterfall loop potential becomes

$$\lim_{x\to 1}\left(V^{(\phi)}_{\rm loop}\right)' \approx \frac{\lambda^4 v^3 \log(2)}{4\sqrt{2}\pi^2} \tag{58}$$

which is to be compared with (54). The waterfall field contribution dominates the one-loop potential for

$$\mathcal{M}^2 < \frac{4\log(2)(1+\delta_m)m^3}{\lambda v^7} \approx \begin{cases} 6.9 \times 10^{12}\, m^3 \lambda^{-10/3}, & \delta_m \ll 1 \\ 3.4 \times 10^7\, m^2 \lambda^{-4/3}, & \delta_m \gg 1 \end{cases} \tag{59}$$

Small $m < 4.9 \times 10^{-6}\lambda^2$ corresponds to the large $\delta_m > 1$ regime. This has to be combined with $m < 1.2 \times 10^{-2}\lambda^{4/3}$ from (42) and

$$\mathcal{M} > \begin{cases} 8.3 \times 10^{-8}\, m^{-1}\lambda^{10/3}, & \delta_m \ll 1 \\ 3.4 \times 10^3\, m\lambda^{-2/3}, & \delta_m \gg 1 \end{cases} \tag{60}$$

from (45). The results for $\lambda = 10^{-4}$ are shown in figure 1b. We see that for smaller couplings the modulus stabilisation scale needs to be larger than the Hubble scale during inflation. E.g. for $\lambda = 10^{-4}$ the inflationary scale is $H_* \approx 10^{-10}$, and $\mathcal{M} > H_*$ in all of parameter space for successful inflation. This contrasts with the situation for larger couplings, as we saw in the previous subsection.

6. Conclusions

The flatness of the inflationary potential in SUGRA models is typically spoilt by corrections coming from supersymmetry breaking. Ironically enough, the vacuum energy which drives inflation breaks SUSY spontaneously, and so gives soft corrections to the inflaton; this is the well-known η-problem. Introducing a shift symmetry for the inflaton will protect the inflation sector from itself, and remove the problem. However there will still be corrections coming from other sectors of the full theory, which can also disrupt inflation. In this paper we studied the effects of a moduli stabilisation sector on a F-term SUGRA hybrid inflation model.

We considered both a KKLT-like moduli stabilisation scheme, in which there is only one scale in the potential so $m_T \sim m_{3/2}$, as well as a fine-tuned two-scale KL-like set-up with $m_T \gg m_{3/2}$. In the KKLT set-up, requiring the modulus to be fixed during inflation raises the scale of the modulus potential, and as a result the soft corrections to both the inflaton slope and the waterfall field masses are too large for inflation to take place. This problem is circumvented in the KL set-up where the gravitino mass, and consequently the corrections to the inflationary potential, can be tuned arbitrarily small.

One would be inclined to conclude that KL moduli stabilisation can be combined almost effortlessly with inflation. But this is not true. The above conclusions only hold in the limit that the modulus field remains fixed during inflation. Although this seems like a good approximation, as the displacement of the modulus minimum during inflation is indeed small, the correction to the flat inflaton potential is nevertheless large. In fact, it gives $\eta \approx -3$, and thus no slow-roll inflation. This analysis shows that it is important to take the dynamics of all fields during inflation into account, otherwise crucial effects may be missed.

We have proposed a way to solve all of the above problems, and successfully combine F-term hybrid inflation with moduli stabilisation. The idea is to combine the modulus and inflaton sectors not by adding their respective superpotentials, as is usually done, but by adding their respective Kähler functions $G = K + \ln |W|^2$ instead. Adding Kähler functions corresponds to adding Kähler potentials and multiplying superpotentials. This way of combining sectors greatly reduces their interactions. In particular, for the case of combining inflation with a modulus sector, it greatly reduces the displacement of the modulus during inflation. Consequently the correction to the inflationary potential is harmlessly small. For the fine-tuned two-scale KL set-up, or for a one-scale KKLT set-up with a fine-tuned mass scale, the corrections to the inflaton slope and waterfall masses are small as well. Hence we indeed succeeded in constructing a successful model of inflation in the presence of moduli.

Even when multiplying superpotentials, there are still some constraints on the moduli sector parameters for viable inflation. The graviton mass should be small enough to suppress the moduli corrections during inflation. The modulus mass needs to be heavy and non-tachyonic during inflation to remain stabilised. Finally the loop potential should be dominated by the contribution of the waterfall fields rather than by the modulus contribution. Nevertheless, there is still a large region of gravitino and modulus mass scales for which inflation works, and the inflationary predictions are nearly indistinguishable from the global SUSY model in the absence of moduli fields.

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